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Understanding Pre-service Teachers Mathematical Concepts Representation for Problem Posing

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Abstract

The purpose of the study is to apply the structure of problem posing as an instructional scaffold in posing problems. The conditions and explorations used by the pre-service teachers during the process of posing reflects how they manage previously learned concepts in mathematics. Results showed how participants posed textbook types of problems, utilized implicit assumptions, compare unrelated concepts, and insufficient information about the problem goal or the sentence structure. The implementation of the structured scheme using Pythagorean Theorem revealed that the instructional scaffold only serves as a "map" rather than a reflective guide so that posing of problems can be systematic and meaningful. Compartmentalization of concepts was a dominant conditions and actions used during the exploration and mainly focused on processing within the same area of representation. Integration of different concepts used during the manipulations of the Pythagorean Theorem reflects a lack of interrelatedness among the elements within the larger structure. Thus, pre-service teachers concerns mostly on how to complete the scheme. The algebraic and geometric representations of Pythagorean Theorem were treated independently rather than treating how the elements interrelates to enable them to function together. Hence, challenging a new concept to replace the existing concept was not successful. Thus, the structured scheme of problem posing suggests an important pedagogical role in understanding the mathematical knowledge of the pre-service teachers.

Keywords: problem posing, structured scheme, mathematical explorations, meaningful relations

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1.0 Introduction

Several decades ago mathematicians (e.g. Silver, 1994; Brown and Walter, 1983; Freudenthal, 1973; Polya, 1957), recognize the centrality of problem posing in the mathematics education. Also, pedagogical and curricular reforms for school mathematics (e.g. Ministry of Education of China, 2011; AAMT, 2002; NCTM, 2000 and 1989) acknowledge the role of students generated problems in school. This movement calls for including problem posing into school curriculum. Silver (1994) refers to "problem posing" to both the generation of new problems and the re-formulation, of the given problems, which can occur before, during or after the solving process. He further classified the situation based on the cognitive activity involved such as: (a) Pre-solution posing- *generation of problems based on the presented stimulus*; (b) within- solution posing- *during the process of solving the posing of questions occur*; and (c) post-solution posing- *after engaging in problem solving activity either to redirect the goals or conditions of the solved problem to generate new problem*. These presented stimuli can be any mathematical topic, idea or situations. Thus the generation of these questions or problems are either results of the situation where it involves an exploration for clarification, problematic situation -about unexplored ideas that needs answer, or an extension of the original idea and generalization of related mathematical concepts. On the other hand, situations in which problem posing task are organized can be distinguished as structured, semi-structure or free (Stoyanova, 1998). A structured task requires the poser to generate questions based on a given specific problem and its solution; a semi-structured task requires the poser to pose a question by exploring the given situation identifying meaningful relations by applying their mathematical skills, knowledge, and experiences; and the situation is free when the poser is asked to generate a question based on a given, contrived or naturalistic situation. In this study, the posing of problems is based on pre-solution, within solution, and post solution posing which utilizes a structured situation.

Intensive works on problems posing that deals on a structured situation was first considered by Brown and Walter (2005, 1990 and 1983) who proposed the well-known "what-if-not" strategy. This method of posing question or problem is associated with Polya's "looking back" stage of problem solving (Cai et al., 2015). The "what-if-not" strategy has the following stages: choosing of a starting point, listing of attributes, "what-if-not-ing", question asking or problem posing, and analyzing the problem. Brown and Walter (2005) argues that although these stages are presented in a linear fashion, it should not be applied in that manner to maximize the benefits of using such strategy.

Usefulness of the strategy was observed by Lavy and Bershadsky (2003) when they used the "what-if-not" strategy in a geometry problem situation. The strategy was conceptualized from teachers' notion about the teaching of Geometry to be difficult. The study was conducted to pre-service teachers to examine the thinking process in the crafting of a geometry problems related the given problem situation. Several occasions students used manipulation of the given, i.e. changing specific numerical data type, manipulation of specific data from the given, during the process of changing the elements within the process of problem posing. This kind of manipulation was considered as "cosmetic changes". Overall results show students' deepened understanding of the geometry concepts. Manipulation and exploration were also the ideas used by Crespo and Sinclair (2008) prior to problem posing activity as forms of intervention in determining the mathematical quality of a problem. The use of manipulation and exploration was based on their initial investigation, where results show that posing of high quality problems requires participants' experience of a problematic situation. This kind of intervention leads to an improved posing of mathematically richer problems of the pre-service teacher-participants. Both studies focus on the thinking

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process and the problematic situations which are used to imply the mathematical knowledge of the pre-service teachers.

Studies on the influence of teachers' beliefs about mathematics that influence their posing of problems in the classroom are well documented (e.g. Livy et al., 2016; Tutak, 2009; Ball et al., 2001; Borko & Putnam, 1996). Teachers pose questions with a pre-conceived agenda, where the purpose of asking is not to generate a meaningful concept, but instead to lay down the trail for the students to follow the rules and procedures they preferred (Brown and Walter, 2005). Posing of problems focuses on the memorization of rules and procedures (Henningsen & Stein, 1997). Thus, there is a need to consider the mathematical experiences of the teachers. Pre-service teacher education program is considered the first formal stage for teachers' development of pedagogical and mathematical knowledge when they transform from expert students into novice teachers in the field. Therefore, it is important to consider the pre-service teacher education preparation program.

In this study, the use of "what-if-not" strategy is considered (Brown and Walter, 2005; Lavy and Bershadsky, 2003) so that the posing of problems becomes systematic and use exploration and the acknowledgement of criteria of a good mathematical problem (Crespo and Sinclair, 2008) during the determination of the meaningful situation or creation of a conjecture (Figure 1). Reports of different studies highlight and suggest the inclusion of students generated activities in addition to having students solve pre-formulated problems, but with, no definite framework on how this problem posing be implemented in the classroom settings. This study focuses on two concepts: structure and exploration. Since these concepts are dealt separately in the growing literature of problem posing, there is an attempt to integrate the two concepts by introducing the structured scheme where explorations were embedded in the scheme. Using the structured scheme, the focus is on issues referring to the relationships of the following: type of questions or problems posed, the choice of exploration which reflects student's mathematical experiences and choice of a meaningful relation or the coming up of a conjecture.

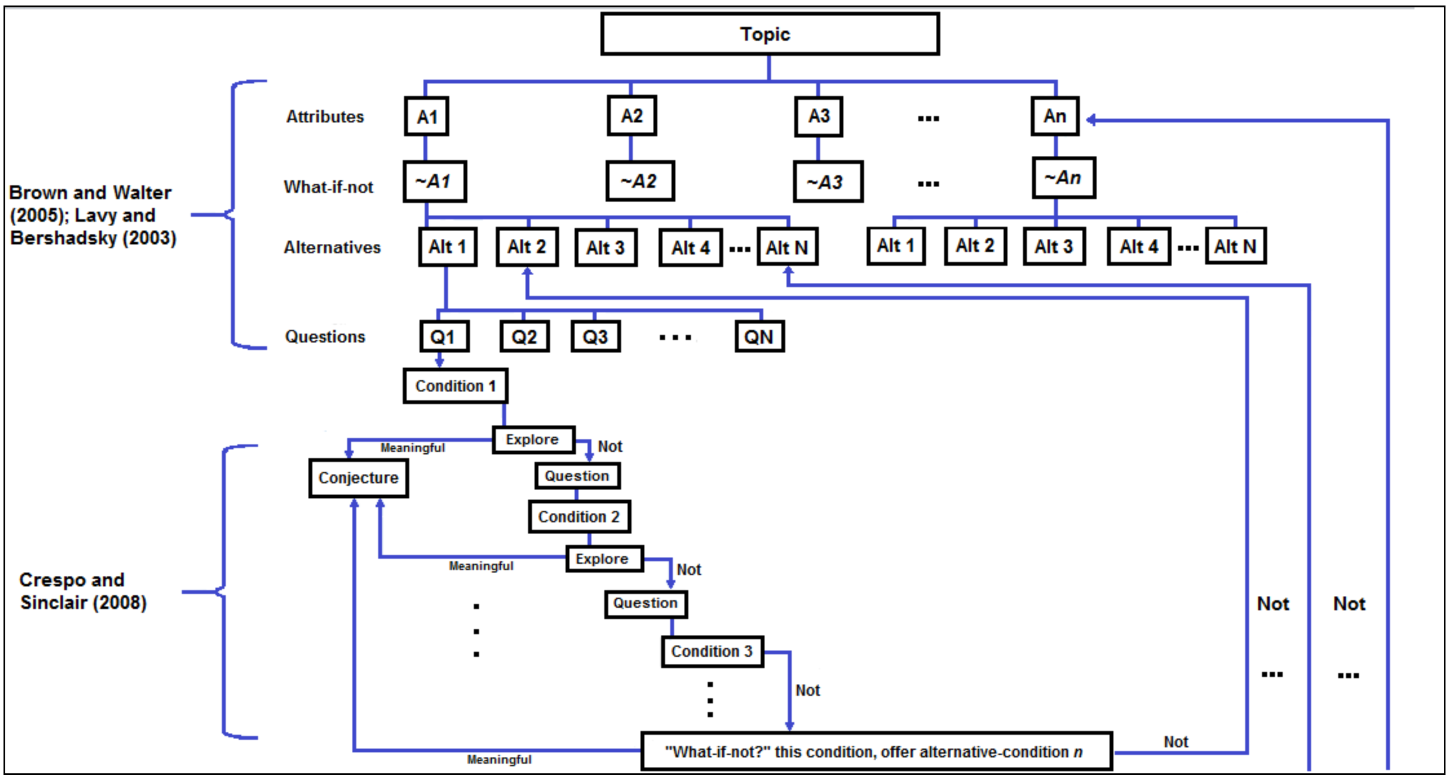


Figure 1. Schematic diagram of the structured scheme

2.0 Research Methodology

Participants

The participants of the study consist of 15 students who are enrolled in "Exploring, Investigating and Modelling Mathematics for Secondary Teaching" course in a state college in the Philippines. They are the only students allowed to continue in the course after passing the evaluation. Qualifying committee based their judgment from the written examination and qualitative evaluation in both major subjects and non-major subjects. They have already took subjects in their previous semesters that include: (1) major subjects - Fundamentals of Mathematics, Fundamentals of Statistics, History of Mathematics, Contemporary Mathematics and Elementary Number theory; (2) special professional education courses; and (3) electives courses.

The course is offered in the second semester of the academic year 2016-2017, the instructor was also the corresponding author. The problem posing was offered as a means of instruction using the developed structure (Figure 1). During discussion, the focus was on the development of a problem or question that can lead to a meaningful relation or the coming up of a conjecture. This will help increase the variation of sources of problems, not only textbook, the internet or other sources, but from their own experience of a problematic situation.

Instruments

The main sources of the data include: (1) the weekly log journal; (2) a summary of their work; and (3) field notes of the instructor. We decided to use these sources of data in the analysis because others have successfully used this kind of strategy. For example, the successful utilization of writing data of Hall et al., (1989) in uncovering cognitive process information about mathematical problem solving lead Silver et al., (1996) to use pen-and-paper data from their study participant instead of using interview data in their analysis of uncovering cognitive process information in problem posing. Although these sources of information are all in written form, they offered different perspectives about the mathematics knowledge and experiences of the pre-service teachers.

The weekly log journal details the kind of exploration they used (e.g. graphing, picture representations, numerical data gathered through a spread sheet, or other software, case by case manipulation using their pen, etc.), the attributes, the questions, variation of the conditions in the exploration, and the meaningful relation they obtain for the concept discussed within that week. The questions can occur in the beginning after the listing of the attributes, in the process of exploration or in the varying of a condition so that a meaningful condition or a conjecture is obtained. The problem posing as a strategy particularly, the kind of questions they posed, the choice and kind of exploration, the varying of the conditions reflects their mathematical knowledge. The choice of a Geometric Situation such as the Pythagorean Theorem is especially chosen as the topic because the concept is very rich of models that are already known and that these topics are part of the topics in the Grade 7 to Grade 10. Aside from the fact that course deals with topics included in the teaching of secondary mathematics.

Data Gathering

At the first week of the term for the "Exploring, Investigating and Modelling mathematics for Secondary Teaching" course, the students are asked to pose questions to and present it during class sessions. The purpose of the activity, the introduction of mathematics problem posing and solving text, is to let them experience the kind of strategy used in the past in dealing with mathematical problems. It is important to note that getting an appropriate problem was not the main objective of the activity, instead the activity serves as a reflection of how the mathematician deals with problem solving and develops problems in the past.

In the third and fourth week, they were introduced to a structure of problem posing (Figure 1). Using the scheme, (1) students are allowed to list all the attributes (internally or externally related to the given mathematical concept or object). From the given attributes, (2) they asked "what-if-not" and offer an alternative. They selected one attribute, or all the attributes they listed and then give an alternative. The alternative is either a situation, a condition, a characteristic of the mathematical object that is not present. (3) The asking or the posing of a question from the given attributes or from the alternative, the focus of the question is either to answer a question that lead to a new idea or to observe relations from original idea of the mathematical object. (4) The exploration stage includes: the manipulation, the making of observation, or the coming of a conjecture. This stage allows the students to use concrete materials, computer simulation software, calculators or any device, the purpose of this is to generate a meaningful relation. Participants are guided in obtaining a meaningful relations using the classification of a mathematically interesting problem scheme developed by Crespo and Sinclair (2008) such as *Fruitfulness, Visual Appeal, Surprise, Simplicity and Novelty*.

In the exploration stage, participants are allowed to explore a mathematical concept by representing the mathematical object using the following: (a) concrete objects (e.g. cans, woods, geoboard, and plastic tangram); (b) free software (e.g., Mathworld Wolfram, R, Geogebra, etc.). The purpose of using these platforms is for finding patterns, visualization and representations of the concept. Note that these different platforms were offered to students as their choice, it was not imposed to them, rather they are the ones to decide which one is more useful during the period of exploration. Following the structure is an important step in introducing to the participants to the problem posing strategy since they do not have background in posing problem. This would ease the difficulty that they might encounter so that problem posing is traceable and systematic. From week 6 to week 10, discussion was focussed on the following topics: Pythagorean Theorem, Arithmetic Mean, Fibonacci sequence, Lucas Sequence and the Generalized Fibonacci-Lucas Sequence. However, this paper only covers the Pythagorean Theorem. Their exploration, questions and problems, and results are all reflected in their log journal. The instructor weekly monitors the journal entry. There are also instances that students have to work on a certain mathematical concept during the class sessions, where they present their work on the board and others during the class participations where they were asked to make comments.

Data Coding

The pre-problem posing activity prior to the introduction of the developed scheme serves only as a background knowledge about the participants and the baseline data about the kind of problems posed by our students when compared with previous study of Silver and Cai (1996) and Crespo and Sinclair (2008). The purpose of the first activity is to determine the types of the problems they generate prior to the introduction of the structured scheme since they have not encountered the strategy in their previous years. Participants are asked to pose at most three problems from the given situation. The problem situation and the tool used in classifying problems is similar to the ones presented by Silver and Cai (1996).

On the second part of the analysis, the exploration particularly focused on the Pythagorean Theorem, how the conditions are used determines the mathematical knowledge of the pre-service secondary teachers through the lenses of representations (Duval, 1999). How the conditions are utilized to produce a meaningful relation is treated whether it involves the processing representation or conversion representation. When involvement of the process of conversion is observed fluently, this means that better coordination of registers was brought into play, hence a reflection of a good mathematical knowledge. Lastly, themes that emerge during the whole process is analyzed and classified.

3.0 Results

The main results are divided in three subsections such as: results of the pre-problem posing stage, results of the structured scheme, and themes that emerge throughout the presentation of the scheme. During the pre-problem posing stage, pre-service teachers were not given any explicit guide on how to generate a question, instead they were given a general instruction to pose a problem based on the given situation. After the pre-problem posing, their problems were classified as statements, nonmathematical, and mathematical. During the introduction of the structured scheme, students were given a sample materials on how to follow the structure. These materials serve as anchors which include: sample works, activities on how to proceed using the structure, and exemplars.

Results of the Pre-Problem Posing Stage

To learn about the problem posing of the group of pre-service teachers prior to the introduction of the structured scheme of problem posing, we use the task presented by Silver and Cai (1996). We wanted to compare our students with their prospective teachers, our participants did not have any encounter with problem posing strategy in the past.

Problem situations presented prior the introduction of the problem posing strategy.

Jerome, Elliot, and Arturo took turns driving home from a trip. Alvin drove 80 miles more than Ellie. Ellie drove twice as many miles as Jake. Jake drove 50 miles.

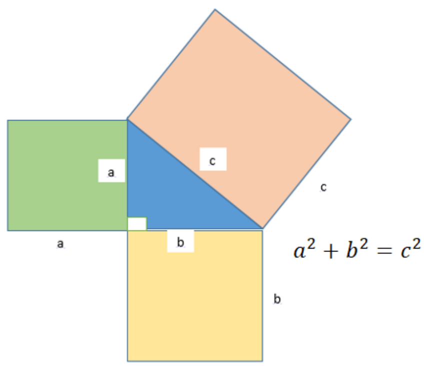
This task was used for middle-school students of Crespo and Sinclair (2008) with their prospective elementary teachers. In our study, we used this in the pre-service secondary teachers that are in the sophomore years. They generated a total of 54 problems. We further classified them into statements, non-mathematical, and mathematical. There were 3 statements, 21 non-mathematical questions and 20 mathematical questions. Several non-mathematical statements emerged. These problems were generated by the students and were lifted verbatim in this section to illustrate an audit trail of the responses. For example, *"How fast Arturo compared to Jerome?" "How many hours in 80 miles?"* These problems seems difficult to understand due to erroneous construction or inaccurate way of asking but were presented here for illustration. Other problems created by the pre-service teachers are illustrated below listed in descending frequency. Although some problems contains errors and other grammatical issues but they are presented here for audit trail purpose and descriptions of their work and not to correct.

1. How many miles did Elliot drive? (6)
2. How many miles did Jerome drive? (3)
3. How many miles do they drive to arrive home from a trip? (2)
4. How many miles did Jerome, Elliot and Arturo drive from a trip going home? (2)
5. How many miles did Elliot drive and how far are they from home? (1)
6. How far are they from home? (1)
7. How far did Jerome, Elliot, and Arturo drive from their trip? (1)
8. How far did they take driving home? (1)
9. How far did they take going/driving home? (1)
10. How many miles did the 3 boys spend in driving home from a trip? (1)
11. How many miles did they drive? (1)
12. How many miles did they drive from a trip to their home? (1)
13. What is the total miles of Jerome, Elliot and Arturo drove? (1)
14. Who took turns the longest driving home from a trip? (1)
15. How many miles did Elliot drive as many as Jerome? (1)
16. How many miles did Elliot drive when she drove twice as many miles as Jerome? (1)
17. If Arturo drove 80 miles and Elliot drove twice as many miles as take. How many did Elliot drive? (1)
18. If Arturo drove with a speed of 40 miles per hour, Elliot with a speed of 25 miles per hour while Jerome drove 10 miles per hour. Who arrived first at home? (1)
19. If Jerome drove 25 miles, then how many miles did Elliot drive going to their home? (1)
20. If Jerome drove 80 miles, then how many miles did Elliot drive? (1)

The analysis of the problems revealed that the majority of the problems posted by our participants are considered in the lowest classification of linguistic complexity- "assignment" problems (problems 1 – 14 in the above list) similar to the kinds of problems generated by the middle-school students in Crespo and Sinclair's study. These problems focused either in group (problems 3, 4 and 6 – 13) or individual (problems 1 and 2) miles covered by the characters in the problem during the trip. Only one problem is considered in the more complex "relational" problem (problem 15), and five problems which are considered "conditional" problem (problem 16 - 20). There are more number of problem variations in our study (20 problems) than in Crespo and Sinclair (8 problems) but these differences are only about how the problems are framed and not the problem goal. Even though participants in our study did not have any background about problem posing, the problems they generated are similar to the middle-school prospective teachers of Crespo and Sinclair.

Results of the Structured Scheme for Problem Posing

Participants followed the scheme and posed different mathematical questions. Figure 2 was used during the investigation. A total of 105 attributes listed by the pre-service secondary teachers from these only 45 were of different types. Nineteen alternatives were offered, from these alternatives 10 were unique and focused on different aspects. Moreover, 30 questions were generated form the task, which they formed before, during or after the exploration, which consisted only of 13 different types. Most of the participants were able to follow the developed structured scheme. However, 4 participants were not able to successfully follow the scheme or could follow but misused some mathematical concepts in the process of exploration. For this reason, their works were ecluded from further analysis. For example, the three of them listed the attributes, offered an alternative, and asked a question but could not proceed to the exploration stage, while another participant could follow the scheme up to the exploration stage, however, during the exploration stage, the offered alternative to the numbers to replace the measures of the sides of the triangle in the Pythagorean Theorem are negative numbers.



The equation and the geometric figure shown is a representation of the Pythagorean Theorem.
As you work out, follow the structured scheme in Problem posing.

Figure 2. The Pythagorean Theorem

The kinds of attributes

The attributes provided by the participants of the study are classified and divided into three kinds. Attributes are classified according to the attributes that deals with the equation, the geometric figure and those comparing either elements of the equations or elements of the geometric figure. Table 1, shows the attributes of the equation where most of the numbers are about the variables and about operations is the least in number. This interest may be attributed to the previous semester encounter of participants in courses like Fundamentals of Mathematics and Contemporary Mathematics which are dealing mostly with equations. Meanwhile, geometric attributes that focused on the parts rather the whole geometric figure that deals with the triangles, squares, sides and angles are shown in Table 2. This kind of lists shows how they utilized the previous subjects in mathematics influenced their created attributes. The geometric representation focused on the individual compartmentalization of topics rather than generalizing and making connections of concepts.

In comparison attributes (Table 3), there was no comparison made between geometric figure and other concepts found in other areas of mathematics. They mostly focused on the geometric object, figure to figure, and equation comparison. They never went out of the context of the given concept; instead, they only accept the given . Hence, mostly working on the same register of information, in the context of the given situation.

The Types of Alternatives Offered

The second level of the scheme is to give an alternative of the Pythagorean Theorem. They are classified according to the kind of changes the participants offered. One kind of change is called *processing* which refers to a change or modification of the original forms but within the same register. For example, a change of figure from square to rectangle, a change of number form used in the original equation which is real number to

Table 1. Attributes about the equation

	Equation	
Variables	**Operation**	**Exponents**
b is less than *c*.	The operation is addition	The exponent are the same.
c is greater than *a*.	It involves addition	The exponent in the formula is 2.
c is the hypotenuse of the triangle	The operation is plus	The exponent is 2.
c is the largest side of the triangle	The equation involves addition.	The exponent is a natural number.
The variables are raised to the 2nd power	There is an equal sign	The exponent is positive integer.
There are three letters *a*, *b*, *c*.		The exponent of *a*, *b* and *c* is 2.
There are three variables .		
x, *y*, *z* are positive integers		
a and *b* are the legs of the triangle		

Table 2. Attributes about the geometric figure

Geometric Figure	
Whole	**Parts**
x^2, y^2 and z^2 are areas of a square, like in the figure.	**Triangle**
	It has one triangle.
The figure has a center figure a right triangle.	The triangle is a right triangle.
	There is a right triangle.
The figure is two dimensional.	**Squares**
	It involves 3 squares.
	The drawing has three squares.
	It involves square.
	There are 3 squares.
	Sides
	It has three sides.
	The sides involves square.
	Angles
	It has a right angle.

complex number. This change is within the register of quadrilaterals of polygons and the latter is the change of sets into another sets. While another classification is called the *conversion*, this refers to a change or modification of figure or number to different forms, a modification or change from one register to another register, or putting additional constraints into the original forms to make a new form. For example, the change of dimensions from the 2-dimensional figure square to a 3-dimensional figure cube, a change of equation to graphical forms. The following are the list of examples of alternatives offered by the participants:

1. It is a rectangle but not a square (9).
2. It is a scalene triangle.
3. It is an isosceles triangle.
4. What if not $x^2 + y^2$, say $\frac{x-y}{r} = z$. For any *r*, positive integer.
5. Suppose it involves division and subtraction, that is, $\frac{a}{b-c} = \left(\frac{a}{b}\right) - \left(\frac{a}{c}\right)$.

Table 3. Attributes comparing relationship

Comparison
The area of the two squares is equal to the largest square.
The sum of the area of the two smaller squares is equal to the area of the bigger square.
The sum of the two squares is equal to the big square.
The squares A and B is equal to the squares C.
The sum of the two squares is equal to the other square.
The triangle in the center is a right triangle.
The right triangle has three square figures adjacent to its sides.
There are three squares in each sides of the triangle.
There is a square in each side of a triangle.
There is one triangle at the center.
The value of x^2 and y^2 is equivalent to z^2.
x^2 and y^2 is equal to z^2.
The measure of the sides of the triangle is not equal.

6. It is a rectangle but not square and numbers are factorial form.
7. It is a rectangle but not a square, and (equations) involves a Logarithm.
8. Suppose the exponent is not 2, say 3, (and the form is a) Rectangular prism.
9. What if not 2, say 3, a cube.
10. The drawing has 3 squares, what if not, let say isosceles triangle.

The figure in the given situation (Figure 2) are squares adjacent to the sides of the right triangle. The items 1 – 5 and 10 are considered processing types of alternatives. The offered alternative of rectangle but not square (number 1 in the list) is a generalization offered by the majority (9 students) since some of the characteristics of rectangles are inherited by squares. Similarly, for the scalene and isosceles triangles (numbers 2 and 3 in the list) are attempts to generalize the kind of triangles that can satisfy the Pythagorean Theorem, what are some restrictions that will be changed from the original form when replaced with alternative form. While numbers 4 and 5, are alternatives to the original equations and totally leading to a different kind of idea that are "analogue" to the Pythagorean Theorem, by changing the form of equations and variables. Numbers 6–9 are examples of alternatives that are considered a conversion. Item 6 and 7 are alternatives about the given equation, offered additional constraints, although a change of square to rectangle is within the same register, it is considered a conversion because of an additional constraint. Items 8 and 9 are conversions types of alternatives about the equation and the geometric figure. A change of the exponent from 2 to 3 and changing the dimension from 2D to 3D. There were instances where the offered alternative was unrelated to the concepts presented, for example, "What if *x*, *y* and *z* are not positive, say negative" "The drawing has three squares, what if not, say isosceles triangle" The first one is meaningless and the other one is unspecified whether the offered alternative is about the drawing of about the number of square. The two alternatives were considered trivial, since no exploration was made by the participants after they offered the alternatives.

The Kind of Questions Posed

The kind of questions are categorized into two broad categories: those that ask about the figure, about the equations, and general about the Pythagorean Theorem. Questions such as the following are the representative of the questions about the figure or the equations: "Could the sum of the area of the two rectangles still be equal to the area of the other rectangle?" "Is it still be true that $a^2 + b^2 = c^2$, where a^2 , b^2 and c^2 are areas of the isosceles triangle?" Can we find numbers than can satisfy the equation?" "What will happen if the widths are divisible by the lengths?" "What are the possible values of the widths or the lengths of each rectangle to sustain the equation $a^2 + b^2 = c^2$?" More than 60% of the questions are about the equations or about the figures asking a number that can satisfy the equation or areas of the rectangle that can satisfy the Pythagorean Theorem when the figure along the sides of the triangle are replaced by rectangle. As can be seen from these examples, these are mostly the kind of questions which they attempted to answer by exploring the concepts involve to uncover a meaningful relation. The figures used instead of square, a rectangle, and the kind of numbers that can be used are very specific. The latter focused is a common problem posed in the textbook or mostly encountered when a teacher ask students to answer about a certain equation in a Geometry class.

Another sets of problems (about 23% of the problems posed) were categorized about the general Pythagorean Theorem. Responses such as the following are representative of the kind: "What pattern emerges for the rectangles that satisfy the formula?" "How to find Pythagorean Triple, given *a*, given *b*, given *c* only?" "Is there a pattern that can solve if only *a*, *b*, and *c* is given?" "What happens to the theorem if the numbers are replaced with factorial forms?" These questions are either asking about a relation, or an extension of the given concept. The remaining sets of questions (about 23%) were judged as inappropriate they are either lacking constraints, question goal, too general or meta-level questions. For example: "Does the outcomes remains the same?" "Find a pattern" "How can I organized the informations?"

The Kinds of Explorations

The exploration stage is classified into context, strategies and intervening conditions. In order complete the explorations stage, participants used the context of Pythagorean triples, types of triangles, forms of numbers (e.g. factorial form, logarithmic form, and natural numbers), parity of numbers (odd, even), intervals of numbers, length-width relationships, and inequalities. While, strategies made by the participants based on the context are the following: invent a formula, made table of values, specify numbers, and enumerate values. Finally pre-service teachers used the following intervening conditions: substitution of values (general forms or specific numbers), observe patterns, and manipulate operations.

The Kind of Meaningful Relations

About 46% of the respondents were able to come up with relations which are about their invented formula or about the conditions they set for manipulations. The following are some of the lists of meaningful relations: (a) "I observed that the observation (2) in explorations 1, 2, and 3 are the same using the formula, $a(bc) + b(ac) = c(2ab)$. Therefore, $a(bc) + b(ac) = c(2ab)$ is applicable to any kinds of triangle and any measurement of the sides of the triangle regardless with the sequence of the variables". (b) "The equation will satisfy if the lengths are Pythagorean triples and the width is a multiple of the length" "Using Logarithm, we can prove the Pythagorean Theorem in the ff. assumptions: (1) The interval is 2: *a, b, c* are all odd or all even; (2) $k = I(1)$; (3) It holds when the values of *a, b, c* are consecutive numbers. **(c)**"For any real numbers *a, b, c* such that *a,b,c* are odd or even or all even which are consecutive, then interval of 2, 3, 4, …, $k = I(1)$, $k = I(2)$, $k = I(3)$, …, *then* $a^2 + b^2 = c^2$". These meaningful relations obtained by the participants are all about finding a number or finding a formula that could satisfy the alternative they offered at the beginning, the questions they asked and the manipulations they used. These reflects the "generic" school mathematics genre of the experience participants' encounter during their previous mathematics. That is, either finding a formula or finding a number. There was even an instance that they never observe that the modified formula created will always hold (meaningful relation (a), because it is an identity, instead adversely concluded that the formula holds for any values. This reflects a non-coordination of the registers of equations and numbers. These suggest that dominant registers function during the activity are confined only within the register of an equation of the registers of values.

4.0 Discussion and Conclusion

We have dealt on the aspect of problem posing that have gained much attention in the growing literature of problem posing: the importance of structures and the use of exploration involve during the process of posing. These two aspects have been treated separately in the literature, here we attempt to integrate each other to form a scheme and identify the kind of mathematical thinking involve during the process of posing up to the generation of an exploration. The type of methods used in this study allows the teachers to successfully implement the method in the class even if they had a first-hand experience of using the structure. The pre-service teachers experience using the approach in a systematic manner showed a traceable posing of a problem. Meanwhile, the dominant processes involved during the exploration describes the kind of participants thinking which are mostly processing of information within the same domain. The exploration stage was drawn on the works of Crespo and Sinclair (2008) which highlights the importance of an authentic experience of the problematic situation. Thus, the posing of problems has a personal connection and also it helps in the variation of the context. Whether it is a problem, a mere statement, and whether constraints provides enough information that guarantees a problem to be solvable, pre-service teachers teacher's did not reflect so long as they can follow the structure. Therefore the process of posing where solving was integrated in the explorations leads only to low level type of problem since most questions are about processing of information.

In comparing and contrasting the meaningful relations, pre-service teachers encountered difficulty of treating the mathematical knowledge. This suggest that when conversion is involve, coordination of registers should brought into play. A reflection of a good mathematical knowledge was not observed, instead the more "generic" school mathematics genre or surface level problem type was dominant. While it is important to consider the pedagogically valuable questions that can elicit mathematical thinking beyond the "generic" problem by which the structure may provide, we found it difficult for the pre-service teachers to do so. Thus, it seems extremely important to aim for future studies that deals at understanding the attitudes of teachers in mathematics towards making meaning of mathematically richer problems that goes beyond rules and value finding, about putting a meaningful connection between different mathematical concepts and beyond compartmentalization of concepts.

References

Australian Association of Mathematics Teacher (AAMT). (2002). *Standards for excellence in teaching mathematics in Australian schools.* Available from: http://www.aamt.edu.au/standards/.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). *Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge.* In V. Richardson (Ed.), Handbook of research on teaching (pp. 433–456). Washington, DC: American Educational Research Association.

Borko, H., & Putnam, R. (1996). *Learning to teach.* In D. Berliner, & R. Calfee (Eds.), Handbook of educational psychology (pp. 673–708). New York: Macmillan.

Brown, S. I., & Walter, M. I. (2005). The art of problem posing (3rd ed.). Hillsdale, NJ: Erlbaum.

Brown, S. I., & Walter, M. I. (1990). *The art of problem posing* (2nd ed.). Hillsdale, NJ: Erlbaum.

Brown, S. I., & Walter, M. I. (1983). *The art of problem posing.* Hillsdale, NJ: Lawrence Erlbaum.

Cai, J., Hwang, S., Chunlian Jiang, C., and Silber,S. (2015) *Problem-posing research in mathematics education: Some answered and unanswered.* In F. M. Singer, N. F. Ellerton, J. Cai (Eds.) Mathematical Problem Posing: From Research to Effective Practice (pp. 3-34). New York: Springer.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education, 11* ,395–415.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, The Netherlands: Kluwer.

Hall, R., Kibler, D., Wenger, E., & Truxaw, C. (1989). Exploring the episodic structure of algebra story problem solving. *Cognition and Instruction, 6* (3), 223–283.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning.

Journal for Research in Mathematics Education, 28, 524–549.

Lavy, I., & Bershadsky, I. (2003). Problem posing via "What if not?" strategy in solid geometry— A case study. *The Journal of Mathematical Behavior, 22(4), 369–387.*

Livy, S. L., Vale, C., & Herbert, S. (2016). Developing Primary Pre-service Teachers' Mathematical Content Knowledge During Practicum Teaching. *Australian Journal of Teacher Education, 41*(2)

Ministry of Education of China. (2011). *Mathematics curriculum standard of compulsory education (2011 version)* [in Chinese]. Beijing, China: Beijing Normal University Press. Retrieved from http://www.moe.gov.cn/publicfi les/ business/htmlfiles/moe/moe_711/201201/xxgk_129268.html

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Polya, G. (1957). *How to solve it.* Princeton, NJ: Princeton University Press.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14* (1),19–28.

Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27 (5), 521–539.*

Silver, E. A., Mamona-Downs, J., Leung, S., & Kenney, P. A. (1996). Posing mathematical problems in a complex task environment: An exploratory study. *Journal for Research in Mathematics Education, 27 (3), 233–309.*

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh & N. F. Ellerton (Eds.), *Research in mathematics education: A contemporary perspective* (pp. 164–185). Perth, Australia, Australia: MASTEC.

Tutak, F. A. (2009). *A study of geometry content knowledge of elementary preservice teachers: The case of quadrilaterals* (Doctoral dissertation. University of Florida).