SOCIETAL OFFENDERS IN PRISON: GLOBAL IN-COUNTRY ANALYSIS BY FRACTALS

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ABSTRACT

The paper examined the relationship between a country's human development index (HDI) and the number of societal offenders in prison (Off. Index) for the country by looking into the variabilities of the data using fractogram or fractal correlation analysis. Result showed that the degree of variability induced by the HDIs tends to decrease the variability of the number incarcerated offenders but number of social offenders vary far greater than suggested by variations in the HDI (like in the case of United States of America). It further revealed that countries belonging to the very high human development index include 1st world countries from the West (with exception to of Asian countries like Singapore, Korea, Japan and Hongkong. Nonetheless, the countries belonging to the low human development index include mostly underdeveloped countries like African nation and some Asian countries like Yemen and Myanmar.

Keywords: human development index, societal offenders in prison, factogram

1.0 Introduction

The prisoners in jails represent the percentage of population who are considered unfit to join society either because they pose danger or because society believes that their seclusion from the normal flow of life would provide them with the necessary time to rehabilitate themselves and thereafter become productive members of the society again. The growing number of prisoners in jails across different countries in the world is alarming (United Nations Office on Drugs and Crime, 2002) and requires a closer look into the phenomenon in order to arrest this trend. This study examines the relationship between a country's human development index (HDI) and the number of societal offenders in jail (Off. Index) for the country. Traditional analysis of this nature strongly veers towards linear (regression) analysis which may be inappropriate for the present case because of the high variability viz. fractality, of the data for the number of offenders in jails. We propose to analyze the phenomenon using fractogram or fractal correlation analysis.

Fractal correlation analysis is a relatively new addition to the arsenal of statistical techniques to analyzing relationships that may not be linear in form. In the past, linear analysis of the relationship between the number of jailed individuals and aspects of human development had been done (Stevens, 2008).The high proportion of prisoners in developed countries may be explained by a range of factors, including better

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funded criminal justice systems, a more strict approach to law and order(e.g. through the use of mandatory sentencing), and a larger gap between the rich and the poor. In non-developed countries, rates of incarceration may be a reflection of a tendency for some.

The Founder of the Human Development Report **MahbubulHaq** (1998) averred *"The basic purpose of development is to enlarge people's choices. In principle, these choices can be infinite and can change over time. People often value achievements that do not show up at all, or not immediately, in income or growth figures: greater access to knowledge, better nutrition and health services, more secure livelihoods, security against crime and physical violence, satisfying leisure hours, political and cultural freedoms and sense of participation in community activities. The objective of development is to create an enabling environment for people to enjoy long, healthy and creative lives."* Thus, the Human Development Index developed over the years covers the main dimensions of education, health and income which are indicated by educational attainment, mean years of schooling , expected years of schooling, life expectancy at birth and gross national income per capita respectively *(Human Development Report Office, UNDP, 2013).*

While it is clear that the rate of incarceration varies inversely as the nation's human development index (HDI), a straight correlation analysis reveals a non-significant correlation coefficient of 0.118 (p-value= 0.138). Not only is this figure misleading in terms of the direction of the relationship, but more importantly it implies that human development has nothing to do with the rate of incarceration or the population of societal offenders. Obviously, a different kind of analysis is required in order to better describe the actual situation obtaining in the various countries. This is the purpose of the present study.

2.0 Basic Concepts in Fractal Statistics

Fractal statistical analysis applies to situations where the mean or first moment does not exist. It also applies to situations where smaller fluctuations dominate the larger ones. Padua (2012) suggested using a power law distribution similar to Pareto's distribution given by:

$$1...\ f(x) = \frac{\lambda - 1}{\theta}\left(\frac{x}{\theta}\right), \lambda > 0, \theta > 0, x \geq \theta$$

where λ is defined as the fractal dimension of X and θ is the smallest (positive) value of the random variable.

The maximum likelihood estimator of λ is

$$2...\ \hat{\lambda} = 1 + \frac{1}{log\left(\frac{x}{\hat{\theta}}\right)}$$

so that each observation contributes to the fragmentation of the support X. Padua (2013) demonstrated that the distribution of the maximum likelihood estimators obey an exponential type of distribution so that both the mean and variance of the fractal dimensions exist.

A device called fractal spectrum or $\lambda(s)$ spectrum was suggested by Padua et al. (2013) to identify locations on the support X where high data roughness or fragmentation occur and where smoothness appear to dominate. The spectrum is defined as:

$$3...\ \lambda(s) = 1 - \frac{log(1-\alpha)}{log\left(\frac{x}{\theta}\right)} = 1 - \frac{log(1-\alpha)}{s}$$

where X_α is the αth percentile of X and $s = log\left(\frac{x}{\theta}\right)$.

Deviations from smoothness indicate the severity of poverty incidence in a given context. A test for deviation from smoothness i.e. $H_0: \lambda = 1$, is suggested in the second paper of Padua (2012) and the reader is referred to the paper as provided in the list of references.

3.0 Research Designs and Methods

The study is descriptive in nature and aims to validate a new procedure for assessing relationships between two variables that do not behave as realizations from a normal distribution but from a power law or fractal distribution Data for the variables are obtained from *http://www.nationmaster. com/graph/cri_pri-crime-prisoners* for the incarceration statistics and *www.epw.in /.../human-development-index-trends- 1980-2012* for the HDI of the countries of the world.

The data obtained are analyzed first by utilizing statistical software to determine the one-dimensional representations of the incarceration rates and the human development indices. This one-dimensional graphical representation was then exported to a fractal software available for free in the net. The fractal software outputs the fractal dimensions of the variables in question. In turn, these fractal dimensions represent the degree to which the variables fragment a smooth straight line.

The two-dimensional scatterplot of the number of offenders incarcerated versus the human development indices of the countries was plotted using the same statistical software. Once again, the plot was exported to the fractal software to obtain the resulting fractal dimension of the two-dimensional graphical representation. This fractal dimension now represents the effect of the ruggedness or roughness of the figures representing Human development on the number of incarcerated individuals. The extent which the roughness of HDI influences the roughness of the number of incarcerated individuals is given by:

$$R2 = 1 - (\lambda_{xy} - 1)^{(\lambda_x \lambda_y)1/2}$$

where:
λ_{xy} = two-dimensional fractal dimension
λ_x = fractal dimension of x
λ_y = fractal dimension of y

4.0 Results and Discussions

Figures 1 and 2 show the one-dimensional plots of HDI and number of incarcerated persons per thousand in various countries while Figure 3 shows the two-dimensional plot of HDI vs number of incarcerated persons. Moreover, table 1 shows the summary of the fractal dimensions computed for the variables of the study.

Variable	Fractal Dimension
X: human Development Index	1.4985
Y: number of incarcerated persons	1.5795
XY: scatterplot	1.2814

$R^2 = 0.908184$: fractal correlation

$R\lambda_{xy} = -0.028$ (p-value= 0.138): correlation of fractal dimensions

$R_{xy} = 0.118$ (p-value=0.734): correlation of original variables
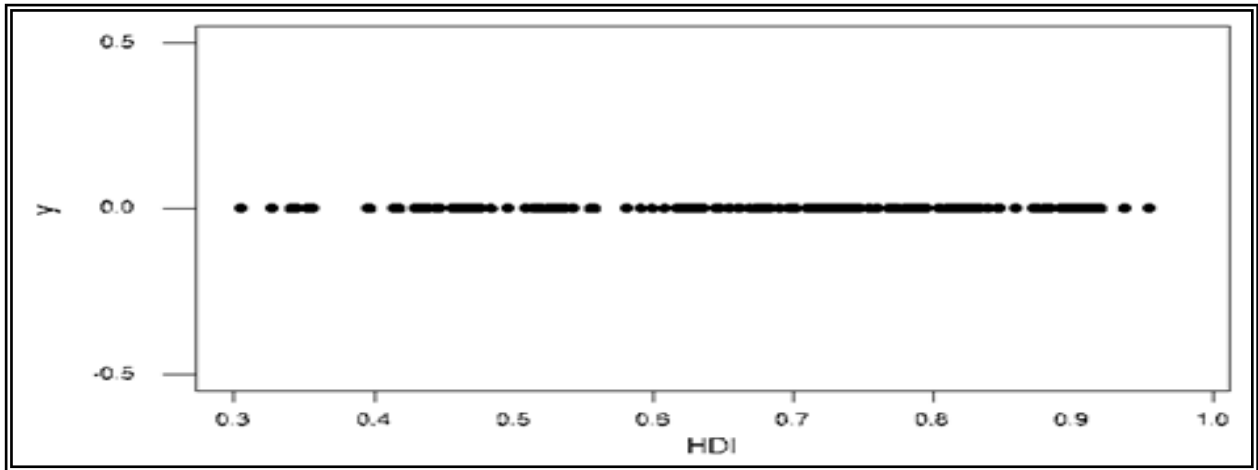
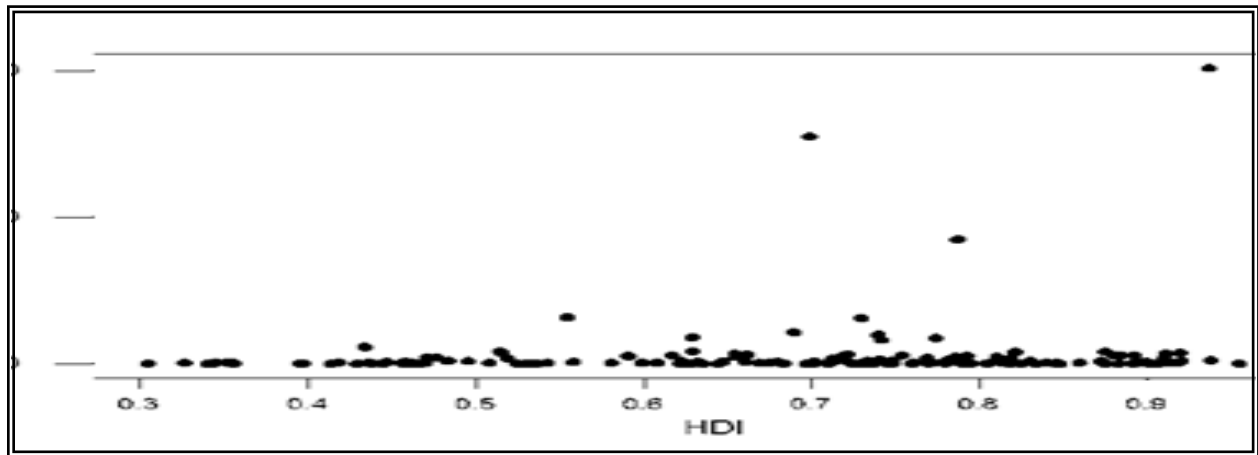Figure 1. One-dimensional plot of human development index of countries

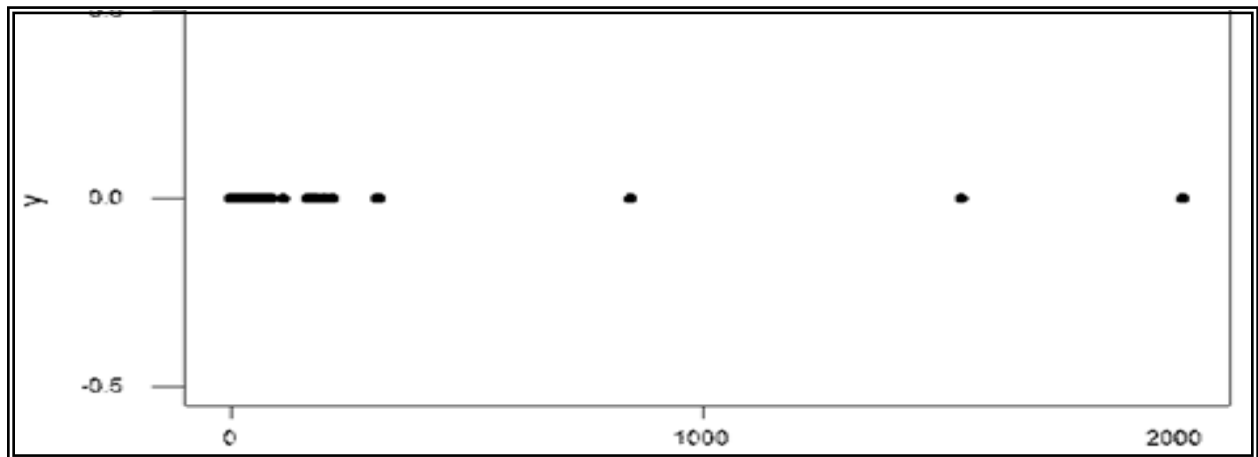Figure 2. One-dimensional plot of offenders' index

Figure 3. Two-dimensional plot of HDI vs offenders' index

The one-dimensional plot reflects the variability of Human Development Indices throughout the world. It is noted that the straight line has been fragmented into four distinct clusters representing low, medium, high and very high human development. The rest of the countries not belonging to the clusters are shown as isolated points on the one-dimensional plot. On the whole, the degree of fragmentation is calculated by its fractal dimension as 1.4985. The deviation statistic which describes the degree to which the data depart from a smooth straight line is roughly 49.85% or 50% deviation from smoothness. Countries belonging to the very high human development include 1st world countries from the West (with the exception of Asian countries like Singapore, Korea, Japan, Hongkong.)The HDI rankings featured above were published in the 2013 Human Development Report: *The Rise of the South: Human Progress in a Diverse World*. On the other hand, the countries belonging to the low human development index include mostly African nations and some Asian countries like Yemen and Myanmar.

The One-Dimensional Plot of Offenders' Index is far more fragmented than the HDI with a fractal dimension of 1.5795 reflecting a smoothness deviation of close to 60% (57.95%). This means that the number of social offenders vary far greater than suggested by the variations in the HDI. This implies too many surprises which cannot be predicted using the traditional methods of analysis. In the case of United States of America, where HDI is reported to be the 3rd best amongst nations, its number of incarcerated offenders is high, in fact ranked number 1.(Human Development Report, 2013). However, in the case of Australia (reported to rank 2nd in terms of Human Development) reported a considerably smaller number of incarcerated offenders. In the case of Norway, ranking 1st in terms of HDI ranked 114 in number of incarcerated offenders. (The Eighth United Nations Survey on Crime Trends and the Operations of Criminal Justice Systems (2002) (United Nations Office on Drugs and Crime, Centre for International Crime Prevention).

The two-dimensional plot reflects the impact of the variability of HDIs to the roughness of the number of incarcerated offenders. The degree of variability induced by the HDIs tends to decrease the variability of the number of incarcerated offenders as shown by the fractal dimension of the two-dimensional plot of 1.2814. Thus, the ruggedness or irregularities of the HDIs of various countries , on the whole, tended to pull down the inherent ruggedness in the reported number of incarcerated offenders viz high human development index induces a corresponding reduction in the number of offenders. It follows that the key to minimizing the occurrence of deviant social behavior is the development of human potentials as reflected in the various dimensions of the human development index.

We note in passing that this conclusions could not have been drawn if the ordinary correlation coefficient (R_{xy}= 0.118 (p-value=0.734) were used because this Pearson measure would have indicated the opposite conclusion. In fact, the correlation of the fractal dimensions ($R_{\lambda_{xy}}$ =-0.028 (p-value= 0.138) in conjunction with the fractal correlation coefficient (R^2 = 0.908184) give a fairer picture of the actual magnitude of the relationship between the two variables.

5.0 Conclusion

The fractal correlation coefficient (fractogram) and the correlation of the fractal dimensions in tandem give a clearer picture of the degree of relationship induced by one highly variant variable on another variable than using the ordinary correlation coefficient. This is illustrated in the case of the relationship between the Human Development Index and the number of incarcerated offenders in the various countries in the world.

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