Fitting a Fractal Distribution on Philippine Seismic Data: 2011-2013

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Abstract

Daily seismic data that occurred in the Philippine area of responsibility from 2011 to April, 2013 were subjected to a fractal analysis. The purpose of the analysis was to fit an appropriate fractal or multifractal distribution on the magnitudes of the observed tremors which can then be used for prediction purposes. Results showed that the mean fractal dimension of the observations was 1.4926. The fractal dimensions computed were observed to obey an exponential distribution with mean $\lambda = 1.4926$. The magnitudes of the seismic observations was fitted to a power-law distribution with fractal dimension 1.4926. Further analysis showed that a multi-fractal distribution can also be fitted to the observations using Bayesian techniques. Multi-fractality can be inferred from the presence of at least three distinct fractal dimensions. Prediction tables were constructed for practical guidance. The authors are currently doing a spatio-temporal analysis of the same data set using fractal statistics.

Keywords: fractal, multifractal, seismic observations, Bayesian analysis

1. INTRODUCTION

Seismic analysis or the study of earthquakes is a very active area of research due to the urgency felt by people worldwide to develop methods for predicting or at least forecasting when such a devastating phenomenon would happen. Advancements in physical sciences, engineering, information technology and mathematical sciences have provided new avenues for predicting the occurrence of earthquakes. One of the most recent tools used by scientists worldwide to predict earthquake occurrence is statistical fractal analysis (Lapenna et al., 2003). This paper attempts to fit a fractal or a multifractal distribution to the Philippine seismic data from 2011 to 2013 with the hope of providing a tool for earthquake prediction which will be useful for disaster risk management and control.

Smalley et al. (1987) pioneered the use of fractal methods in seismic analysis which is reported by Kagan (1994) in a review of experimental evidences for earthquake scale-invariance. In earlier works, a single fractal exponent was used in conjunction with earthquake modeling Lapenna et al., (2001) but later evidences point to the need for multiple fractal exponents to explain the behavior of seismic patterns (Lapenna, (2013). Padua et.al. (2013) developed a test for determining whether a set of observations is monofractal or multifractal. Most of the studies on fractal methods, however, were done in European countries and the United States with little or negligible papers in Asia on the matter.

In the Philippines, fractal methods are hardly, if ever, used in conjunction with the problem of earthquake predictions and seismic analysis. Traditional seismic analysis techniques are still being used whilst at the same time attempting to acquire sophisticated earthquake detection instruments at the PHILVOLCs of the Department of Science and Technology (DOST-Philvolcs, 2013). For instance, time series observations of earthquake occurrences using Discrete Fourier transforms had been tried out in the past (University of California-Los Angeles,

1992) but since then, little progress had been achieved on the matter of seismic analysis and prediction. An analysis of the available seismic data from the agency through fractal methods would be useful for the agency and the general public as it provides new insights into this phenomenon.

2. Statistical Fractals

Statistical fractal observations are relatively new in the discipline of Statistics. While classical statistics depend on the existence of a mean (μ) and variance (σ^2) of a set of normally distributed random observations [$N(\mu,\sigma^2)$], most real- life observations do not possess a mean nor a variance. Random observations that obey a fractal observation are characterized by having smaller values than lesser values repeated at different scales. Such fractal distributions are represented by the power-law probability density function:

1… $f(x) = \frac{\lambda-1}{\theta}\left(\frac{x}{\theta}\right)^{-\lambda}$, $\lambda>1$, $\theta>1$, $x \geq \theta$

where θ is the minimum of the x's and λ is the fractional exponent. The term "fractal "is derived from the fact that the exponent of (1) is fractional.

The cumulative distribution function (cdf) of (1) is given by:

2… $F(x) = \int_\theta^x f(x)\,dx = 1 - \left(\frac{x}{6}\right)^{1-\lambda}$, $\lambda > 1$

If $x_1, x_2, \ldots x_n$ are earthquakes magnitude obeying a fractal distribution (1), then one can compute the probability that an earthquake of magnitude between x_k and x_{k+t} will occur as:

3… $P(x_k < x < x_{k+t}) = F(x_{k+t}) - F(x_k)$

For this reason, fitting a probability model such as (1) to the seismic data in the Philippines would be useful.

In Padua et.al. (2013), observations of $x_1, x_2, \ldots, x_n$ coming from an unknown distribution G(•) are fitted to a fractal distribution with fractal dimensions as follows:

Let $\alpha_i = \frac{i}{n}$, $1 \leq i \leq n-1$ and denote α_k quartile x_{α_k}. It follows that $x_{(\alpha_1)} \leq x_{(\alpha_2)} \leq \cdots \leq x_{(\alpha_n)}$ Since $(x_{(\alpha)}) = \alpha$, it follows from (3) that

4. $\lambda_{(\alpha)} = 1 - \frac{\log(1-\alpha)}{\log\left(\frac{x_{(\alpha)}}{\theta}\right)}$, for all $\alpha \in (0,1)$,

where θ is the observed minimum of the x's. These values of λ approximately exponentially distributed so that the mean exists with an estimator:

5. $\hat{\lambda} = \frac{1}{n-1}\sum_{k=1}^{n-1}\lambda_{(\alpha_k)}$

The authors then proceeded to develop a mono fractality test whose algorithm is given below:

Test Algorithm

(1) Sort $x_1, x_2, \ldots, x_n$ from smallest to highest. Denote the sorted values by $x_{(1)} \leq x_{(2)} \leq \ldots x_{(n)}$.

(2) Assign the weights α_i = i = 1,2,…, n. Remove the last observations (highest observation). Let $\theta = x_{(1)}$. .
(3) Compute $\hat{\lambda}$ based on (4) and (5).
(4) Do a time series plot for, i = 1,2,…, n-1, and regress $\hat{\lambda}_{\alpha_i}$ versus t where t is defined in result 2, $\hat{\lambda}_{\alpha_t} = a + b\,t$:

4.1. If Ho: b = 0, then conclude $\{x_1, \ldots, x_n\}$ came from a power – law distribution with fractal dimension λ;

4.2. If Ho: b = 0 is rejected, then there is a trend and $\{x_1, \ldots, x_n\}$ come from a non – fractal distribution;

4.3. If the plot shows that for $0 < \alpha < \alpha_i$, b = 0, i -1, …, n-1 then there are multiple values of λ, $\lambda_1, \ldots, \lambda_n$ for each segment $0 < \alpha < \alpha_i$, and the data came from a multifractal distribution.

3. Fitting a Fractal Distribution on Philippine Seismic Data

Data from the NET supplied by the Department of Science and Technology-PHILVOLCS on a daily basis from 2011 to to 2013 were used for the study. Approximately data points were obtained both for the magnitude of the seismic signals and inter-event times (2,964 data points). The inter-event times (τ) were manually computed based on the published data from the DOST.

A plot of the earthquake magnitudes (on a Richter scale) is given in figure 1.

The plot shows that earthquakes of magnitudes 5 and above on the Richter scale are few with seismic records cluster around 3 or lower on the Richter scale.

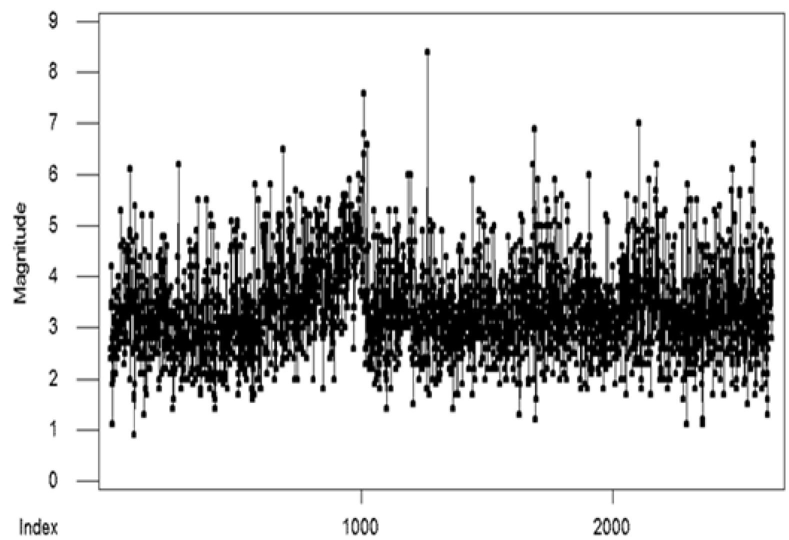

Figure 1. Time series plot of the earthquake magnitudes from most recent records of seismic disturbances in the Philippines

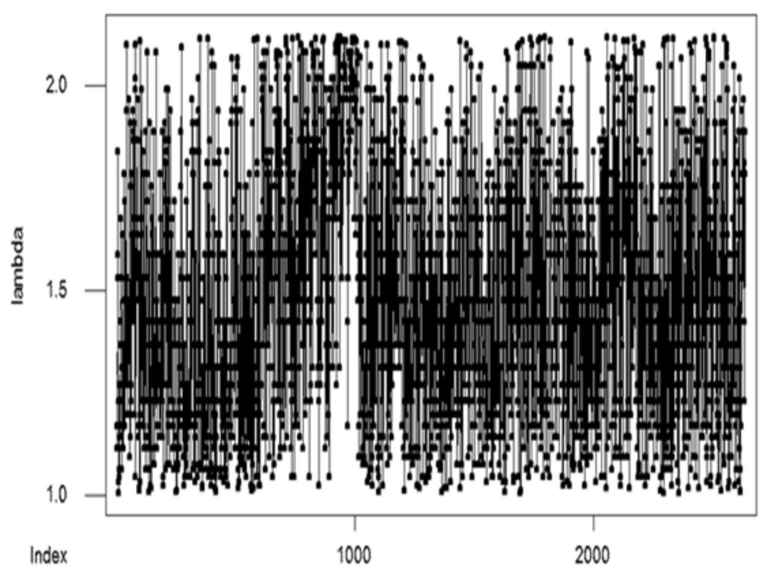

Figure 2. Time series plot of the estimated fractal dimensions from most recent seismic observations

Following Padua et al. (2013) we proceeded to fit a fractal distribution on the quantiles of the data set. The results are shown as a time-series plot of the fractal dimensions as shown in figure 2.

The estimated mean fractal dimension is 1.4926 with a standard deviation of 0.3131. Note that this fractal dimension is larger than the fractal dimension of a normal sequence of random variables. The plot of lambda versus scale is given in Figure 3 while for comparative purposes, we have plotted the fractal dimension against scale for the absolute values of N(0,1) in Figure 4.

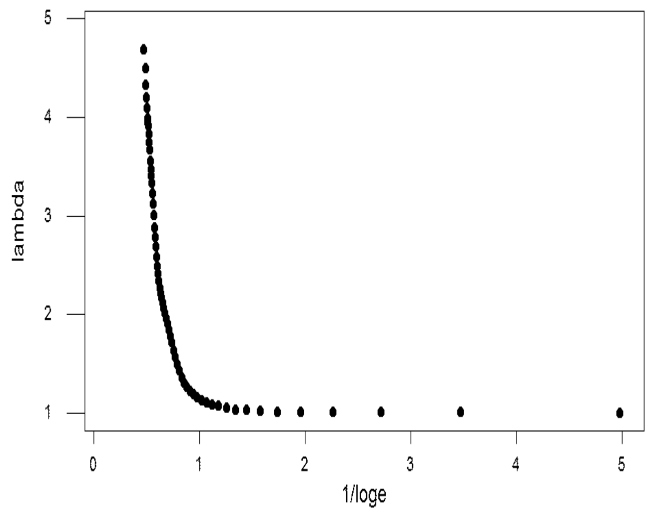

Figure 3. Plot of fractal dimensions versus scale for earthquake dimensions.

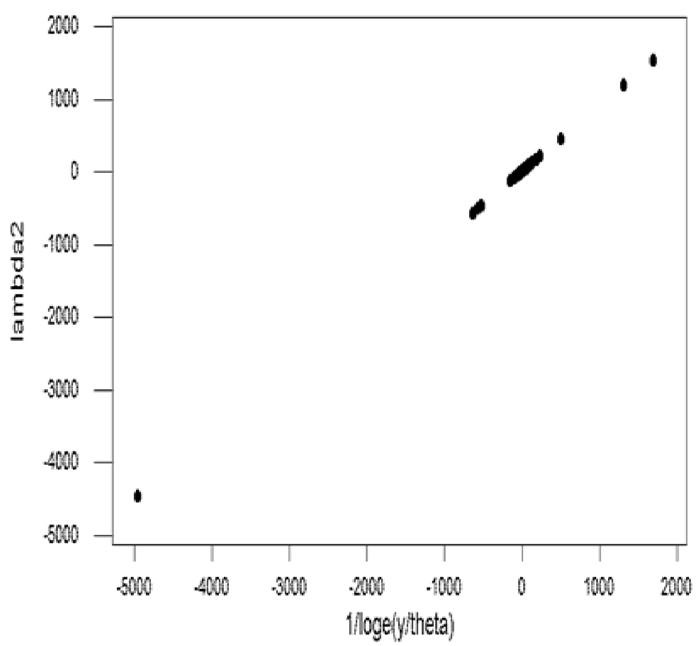

Figure 4. Plot of standard normal fractal dimensions against scale.

The fractal dimensions of the earthquake magnitudes appear to follow an exponential distribution with mean (1.6657). The estimated fractal distribution for the magnitudes of earthquakes in the Philippines is therefore:

Fractal Model for Earthquake Magnitudes in the Philippines:

6. $f(x) = \frac{0.4926}{0.9}\left(\frac{x}{0.9}\right)^{-1.4926}$, x ≥0.9

Using this model, we can estimate the probabilities of occurrence of earthquakes of various magnitudes in the future.

Table 1. Earthquake magnitudes in the Philippines: probability of occurrence.

Range Magnitude of Earthquake (Richter Scale)	Probability of Occurrence	Probability in Percent
0.9 to 1.5	0.121758	12.18%
1.6 to 3.1	0.114621	11.46%
3.2 to 4.7	0.050544	5.05%
4.8 to 6.3	0.030082	3.01%
6.4 higher	0.026107	2.61%

Seismic disturbances of magnitudes 4.5 or higher have the potential to cause from moderate to extreme damage. Table 1 above shows that the probability of such an event happening is approximately 10.67% (or 11%). Table 2 is culled from the US Geological Survey to guide in the interpretation. The earthquake with the largest magnitude of 8.4 in the Richter scale that hit the Philippines, according to Table 2 can cause " Major damage to buildings; structures likely to be destroyed. It will cause moderate to heavy damage to sturdy or earthquake-resistant buildings. Damaging in large areas, will be felt in extremely large regions." Death toll ranges are quite high and will be dependent on the population density of the area hit.

Earthquakes of the magnitude previously described rarely hit the country. The estimated probability of one such occurrence is less than 1% (.0035 or 0.35%) so that in a 365-day year, an earthquake of intensity 8 or higher, will almost certainly not occur (99.65%). Despite the very low probability of occurrence of devastating earthquakes, governments all over the world are still spending time and effort in

Table 2. Guide to interpretation of earthquake effects

Magnitude	**Description**	**Mercalli Intensity**	**Average Earthquake Effects**	**Average frequency of occurrence (estimated)**
Less than 2.0	Micro	I	Micro earthquakes, not felt, or felt rarely by sensitive people. Recorded by seismographs.	Continual/ several million per year
2.0–2.9	Minor	I to II	Felt slightly by some people. No damage to buildings.	Over one million per year
3.0–3.9		II to IV	Often felt by people, but very rarely causes damage. Shaking of indoor objects can be noticeable.	Over 100,000 per year
4.0–4.9	Light	IV to VI	Noticeable shaking of indoor objects and rattling noises. Felt by most people in the affected area. Slightly felt outside. Generally causes none to minimal damage. Moderate to significant damage very unlikely. Some objects may fall off shelves or be knocked over.	10,000 to 15,000 per year

Table 2. cont. Guide to interpretation of earthquake effects

Magnitude	Description	Mercalli Intensity	Average Earthquake Effects	Average frequency of occurrence (estimated)
5.0–5.9	Moderate	VI to VIII	Can cause damage of varying severity to poorly constructed buildings. At most, none to slight damage to all other buildings. Felt by everyone. Casualties range from none to a few.	1,000 to 1,500 per year
6.0–6.9	Strong	VII to X	Damage to many buildings in populated areas. Earthquake-resistant structures survive with slight to moderate damage. Poorly-designed structures receive moderate to severe damage. Felt in wider areas; up to hundreds of miles/kilometers from the epicenter. Damage can be caused far from the epicenter. Strong to violent shaking in epicentral area. Death toll ranges from none to 25,000.	100 to 150 per year
7.0–7.9	Major	VIII or greater	Causes damage to most buildings, some to partially or completely collapse or receive severe damage. Well-designed structures are likely to receive damage. Felt in enormous areas. Death toll ranges from none to 250,000.	10 to 20 per year
8.0–8.9	Great	VIII or Greater	Major damage to buildings, structures likely to be destroyed. It will cause moderate to heavy damage to sturdy or earthquake-resistant buildings. Damaging in large areas, will be felt in extremely large regions. Death toll ranges from 100 to 1 million.	One per year (rarely none, two, or over two per year)
9.0–9.9			Severe damage to all or most buildings with massive destruction. Damage and shaking extends to distant locations. Permanent changes notable in ground topography. Death toll ranges from 1,000 to several million.	One per 5 to 50 years

(Based on U.S. Geological Survey documents.)(^http://earthquake.usgs.gov/earthquakes/recenteqsus/Quakes)

developing more accurate and precise techniques for predicting their occurrence, particularly in the spatio-temporal scale because of the very serious consequences of earthquakes with large magnitudes. Further analysis of the data set, however, revealed that the earthquake magnitudes can be expressed more precisely as multifractal random variables. More specifically, regression runs of lambda versus scale, revealed at least three fractal dimensions: λ_1 =1.0059, λ_2 = 1.4926, λ_3 = 2.1170, and possibly many more. The histogram of the estimated values of the fractal dimensions is shown below and appears to behave like an exponential random variable or possibly a normal random variable:

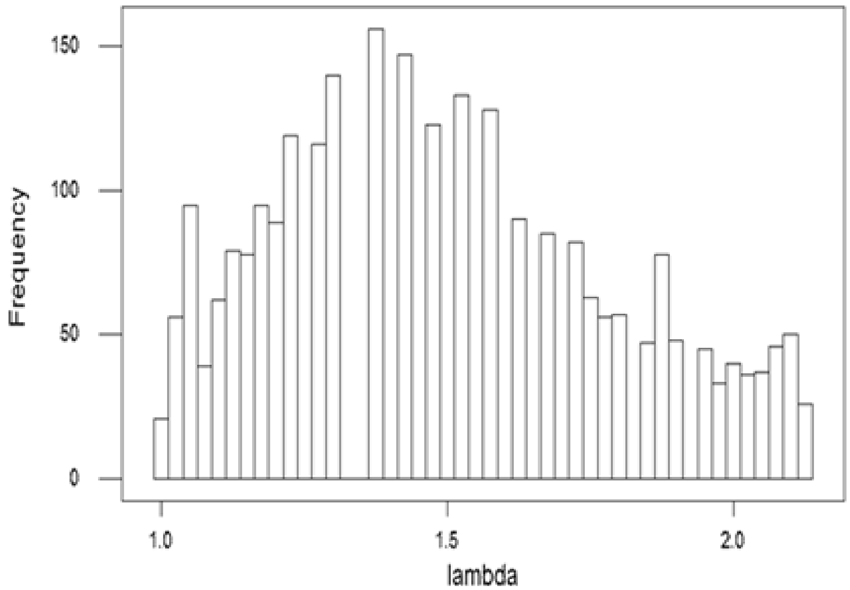

Figure 5. Histogram of the estimated values of the fractal dimensions

A quick test for normality of lambda by the Kolmogorov-Smirnov statistic revealed that in fact the fractal dimensions are not normally distributed.

In fact, it appears that the fractal dimensions behave like exponential random variables with mean 1.4926 and minimum 1.005. The model is:

7. $g(\lambda) = Ae^{-k\lambda}, \lambda > 1,$

where A and k are determined so that g (.) is a probability density function on $\lambda > 1$. After doing the required calculus, we find that:

8. $k = \ln\left(\frac{A}{k}\right)$ or $A = ke^{k}$. This means that (7) becomes:

9. $g(\lambda) = ke^{k}e^{-k\lambda} = ke^{-k(\lambda-1)}\ \lambda > 1.$

The expected value or mean of distribution (9) is $\mu_\lambda = 1 + \frac{1}{k}$. The estimated mean of the fractal dimensions is 1.4926 so the value of k is $k = \frac{1}{0.4926} = 2.030$. We can now take (9) as a *prior* distribution of λ in a Bayes sense to obtain the distribution (10) below:

10. $g(\lambda) = 2.030\ e^{-2.030(\lambda-1)}\ \lambda >$

We can derive the multi-fractal distribution by Baye's theorem:

Estimated Multifractal Distribution of Earthquake Magnitudes: Philippines

$$f(x,\lambda) = f(x/\lambda)g(\lambda) = \left(\frac{x}{0.9}\right)^{-1.4926} (2.030)\ e^{-2.030(\lambda-1)}$$

11. $f(x,\lambda) = 1.111 \frac{0.4926}{0.9}\left(\frac{x}{0.9}\right)^{-1.4926} e^{-2.030(\lambda-1)},\ x \geq 0.9,\ \lambda \geq 1$

Equation (11) can now be used to revise the estimates of the probabilities of occurrence of earthquakes within certain magnitude ranges. The revised estimates are given below using the mean fractal dimension:

Table 3: Revised Estimates Based on a Multifractal Model

Range Magnitude of Earthquake (Richter Scale)	Probability of occurrence	Probability in Percent
0.9 to 1.5	0.09093	9.09%
1.6 to 3.1	0.0856	8.56%
3.2 to 4.7	0.037747	3.77%
4.8 to 6.3	0.022465	2.25%
6.4 higher	0.019497	1.95%

We note that the revised Bayesian estimates of occurrences of various earthquake magnitudes have reduced significantly.

4. CONCLUSIONS

The magnitudes of earthquakes that occurred in the Philippines from January 1, 2011 to April 12, 2013 were found to obey a fractal distribution with fractal dimension $\lambda = 1.492$. More precisely, the earthquake magnitudes consisted of several fractal dimensions and are therefore better modeled in terms of a multifractal distribution. The multifractal distribution is constructed using Bayesian methods where the observed fractal dimension is a priori modeled as an exponential distribution with mean 1.4926. Estimates of occurrence of various earthquake magnitudes are provided and results suggests that potentially damaging earthquakes (magnitude 4.5 above) will occur with more than 10% probability.

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